

## Runaway Dynamics and Supersymmetry Breaking

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### Abstract

Supersymmetric  $SU(N_C)$  gauge theories possess runaway-type superpotentials for  $N_F < N_C$ , where  $N_F$  is the flavor number of massless quarks. We show that the runaway behavior can be stabilized for  $N_F \simeq N_C$  by introducing singlets with the aid of perturbative corrections to the Kähler potential, generating (local) minima of supersymmetry breaking.

# 1 Introduction

It is well known that in a supersymmetric (SUSY) QCD based on an  $SU(N_C)$  gauge theory with  $N_F$  flavors of quarks  $Q$  and antiquarks  $\tilde{Q}$  [1], the dynamically generated superpotential implies a runaway behavior for  $N_F < N_C$ . In this theory we have SUSY-invariant vacua in the limit of meson fields  $|Q\tilde{Q}| \rightarrow \infty$ . This is consistent with Witten index argument for unbroken SUSY [2, 3]. In particular, the Witten index [2] correctly counts the number of SUSY vacua that is determined by adding the mass terms for all the pairs of quarks and antiquarks.

However, the situation is changed if we introduce a singlet field  $S$  and assume a tree level superpotential of the form  $SQ\tilde{Q}$ . This is because one pair of quark and antiquark is always taken massless by shifting the  $S$  field even if we introduce the mass terms for all the pairs of quarks and antiquarks, and the Witten index argument based on the mass deformation does not apply. It seems that this theory can have a SUSY breaking vacuum due to gauge dynamics in principle. Unfortunately, we see, after integrating out quark fields, that we also have a runaway-type superpotential of the singlet  $S$ . Thus, it seems likely that the theory has a runaway potential for the  $S$  boson and the vacuum expectation value (vev) of  $S$  goes to infinity in SUSY vacua. Owing to this runaway behavior, this theory was not considered as a SUSY breaking model<sup>1</sup>.

We here note that there is subtlety in connection with quantum corrections to the Kähler potential. To compute a potential in a supersymmetric theory, we have to know both the superpotential and the Kähler potential of the theory. That is, even if the superpotential is of the runaway type, there is a possibility that the runaway of the singlet  $S$  is stopped through the effects of the Kähler potential. If such a stabilization occurs, then the  $S$  has a nonvanishing  $F$ -term at the potential minimum and SUSY is broken.

In this paper, we show that such a stabilization does indeed occur in a certain parameter region. We restrict our discussion in a weak coupling regime so that the Kähler potential is calculable perturbatively. We find out that the Kähler potential can stop the

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<sup>1</sup>We cannot exclude the possibility that there are SUSY breaking local minima at small vevs of the singlet field  $S$  [4].

runaway of the potential in a class of models with  $N_F \simeq N_C \gg 1$ . As we will see below, this stabilization is due to the anomalous dimension of the  $S$  field, which is determined mainly by the Yukawa coupling of the theory at the leading order in perturbation theory.

## 2 Runaway potential in SUSY QCD

Let us consider a SUSY  $SU(N_C)$  gauge theory with  $N_F$  flavors of quarks  $Q^i$  and antiquarks  $\tilde{Q}_i$  ( $i = 1, 2, \dots, N_F$ ), which belong to the fundamental and the anti-fundamental representations of the  $SU(N_C)$ , respectively. We restrict our discussion in the case of  $N_F < N_C$ . We introduce a singlet chiral multiplet  $S$  and assume a tree-level superpotential

$$W = \lambda S Q^i \tilde{Q}_i. \quad (1)$$

Here, we have omitted the  $SU(N_C)$  indices.

Let us consider a region where the singlet  $S$  has a large vev. The vev of  $S$  gives the mass  $\lambda S$  to the quarks  $Q$  and antiquarks  $\tilde{Q}$  through the superpotential (1). When we integrate out the massive quarks  $Q$  and  $\tilde{Q}$ , the low energy gauge dynamics is described by a pure  $SU(N_C)$  gauge theory with the dynamical scale  $\Lambda_L$  given by<sup>2</sup>

$$\Lambda_L^{3N_C} = (\lambda S)^{N_F} \Lambda^{3N_C - N_F}, \quad (2)$$

where  $\Lambda$  denotes the dynamical scale of the high energy theory.

In the pure  $SU(N_C)$  gauge theory, the gaugino condensation occurs, which generates a superpotential

$$W_{\text{eff}} = N_C \Lambda_L^3 = N_C \Lambda^{3-R} (\lambda S)^R, \quad (3)$$

where  $R$  denotes the ratio  $R = N_F/N_C < 1$ . Thus, if the Kähler potential took a tree level form  $K = |S|^2$ , the potential of  $S$  would be given by

$$V = N_F^2 \left| \Lambda^{3-R} \lambda^R S^{-1+R} \right|^2, \quad (4)$$

which exhibits a runaway behavior with the vev of  $S$  going to infinity.

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<sup>2</sup>For the coefficients of scale matching and dynamically generated superpotentials, we follow Ref.[5].

In reality, we should take into account quantum corrections to the Kähler potential of  $S$ , which can change the above mentioned runaway behavior (as well as the behavior around a small vev of the singlet field  $S$ , which we do not investigate in this paper). The low energy effective Kähler potential is given by  $K_{\text{eff}}(|S|, M)$ , where we adopt a renormalization scheme [6]

$$\ln \frac{\partial^2 K_{\text{eff}}}{\partial S \partial S^*} = \int_{\mu=|S|}^M \tilde{\gamma}_S(\mu) d(\ln \mu), \quad (5)$$

with  $M$  the renormalization point and  $\tilde{\gamma}_S(\mu)$  the corresponding anomalous dimension at the scale  $\mu$ . The potential is obtained as

$$\begin{aligned} V &= N_F^2 \left| \Lambda^{3-R} \lambda^R S^{-1+R} \right|^2 \left( \frac{\partial^2 K_{\text{eff}}}{\partial S \partial S^*} \right)^{-1} \\ &= N_F^2 \left| \Lambda^{3-R} \lambda^R S^{-1+R} \right|^2 \exp \int_{\mu=M}^{|S|} \tilde{\gamma}_S(\mu) d(\ln \mu). \end{aligned} \quad (6)$$

In the next section, we show by a perturbative calculation that the runaway behavior of the potential can indeed be stopped in the theory considered above. We see that the  $S$  is stabilized with its large vev, where the perturbative corrections to the Kähler potential are estimated reliably to dominate.

### 3 Stopping the runaway

Let us consider the theory in the previous section and show that the runaway behavior of the potential can be stopped by the effects of the Kähler potential. We assume that the gauge coupling is so small at the relevant energy scale  $M$  that the perturbative calculations are reliable. We also assume that  $1 - N_F/N_C = 1 - R$  is very small, which can be realized for sufficiently large  $N_F \simeq N_C$ .

The one-loop corrected Kähler potential is given by

$$K_{\text{eff}} \simeq \left( 1 - \frac{N_C N_F |\lambda|^2}{16\pi^2} \ln \left| \frac{S}{eM} \right|^2 \right) |S|^2, \quad (7)$$

which indicates  $\tilde{\gamma}_S \simeq N_C N_F |\lambda|^2 / 8\pi^2$ . The above one-loop result is reliable for  $|S| \sim M$  with the running coupling  $\lambda(M)$  small enough, which enables us to ignore the higher-order corrections.

Using this Kähler potential and the superpotential (3), the potential of  $S$  is determined to be

$$V \simeq \left(1 + \frac{N_C N_F |\lambda|^2}{16\pi^2} \ln \left| \frac{S}{M} \right|^2\right) N_F^2 \left| \Lambda^{3-R} \lambda^R S^{-1+R} \right|^2. \quad (8)$$

Differentiating this potential with respect to  $\ln |S|$ , and taking only the leading terms in  $|\lambda|^2$  and  $1 - R$ , we obtain

$$\frac{\partial V}{\partial \ln |S|} \simeq 2 \left( \frac{N_C N_F |\lambda|^2}{16\pi^2} - 1 + \frac{N_F}{N_C} \right) N_F^2 \left| \Lambda^{3-R} \lambda^R S^{-1+R} \right|^2. \quad (9)$$

The above perturbative results are reliable for  $M \sim |S|$ . Thus we conclude that the potential has SUSY breaking minima for

$$\tilde{\gamma}_S(|S|) \simeq \frac{N_C N_F |\tilde{\lambda}|^2}{8\pi^2} \simeq 2 \left(1 - \frac{N_F}{N_C}\right), \quad (10)$$

with the aid of the running coupling  $\tilde{\lambda} = \lambda(|S|)$ . Note that  $N_F \simeq N_C \gg 1$  must be imposed for the coupling  $\tilde{\lambda}$  to be adequately small. The appearance of the anomalous dimension  $\tilde{\gamma}_S$  is no accident. In fact, this result can be directly derived by means of the expression (6).

The behavior of the potential around the SUSY breaking minima can be seen through that of the couplings. We assume that the gauge coupling is negligibly small compared with the Yukawa coupling  $\lambda$  at high energy. Then, the renormalization group equation of  $\lambda$  is given by

$$M \frac{\partial}{\partial M} |\lambda|^2 \simeq \frac{N_C N_F + 2}{8\pi^2} |\lambda|^4, \quad (11)$$

so that  $|\lambda|^2$  as well as  $\tilde{\gamma}_S$  is an increasing function of  $M$ . This confirms that the potential indeed has the above mentioned minima if we take the coupling  $\lambda$  to satisfy

$$\frac{N_C N_F |\lambda|^2}{8\pi^2} \ll 2 \left(1 - \frac{N_F}{N_C}\right), \quad (12)$$

at a certain high energy scale.

Finally, a few comments are in order. Eq.(11) further implies that  $\lambda$  eventually hits a so-called Landau pole at higher energy, which reveals that our perturbative description does not apply to the regime of much larger  $|S|$ . We also note that we still have SUSY vacua of mesonic runaway directions with only one singlet  $S$ , though such vacua may be eliminated by means of sufficiently many singlets as in Ref.[7].

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